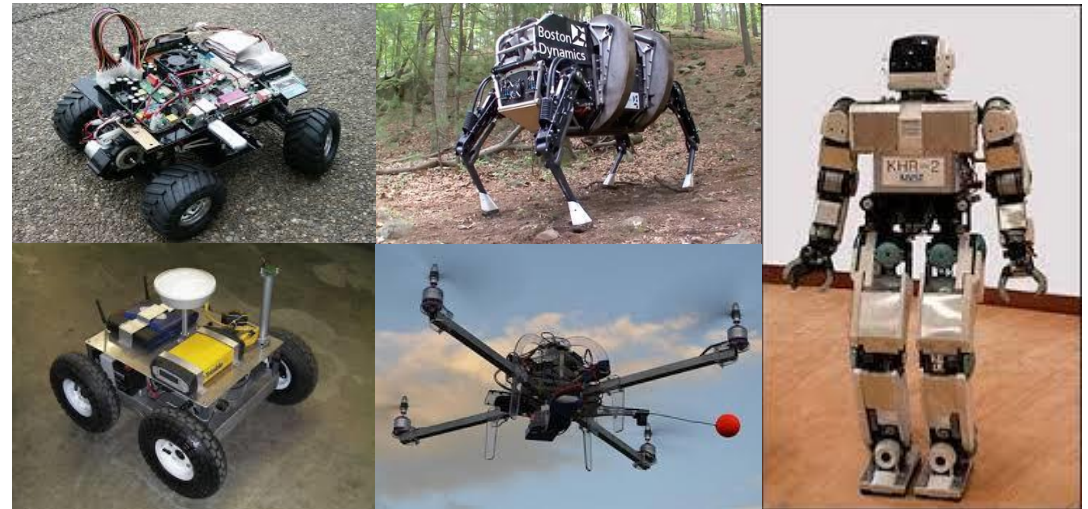# Mobile Robots

MUHAMMAD A. SHEHAB

Mobile Robots: Overview

▶ Mobile robot is a class of robotic systems that has the ability to move around in its environment and is not fixed to one physical location.

▶ A mobile robot needs locomotion mechanisms that enable it to move unbounded throughout its environment.

▶ Examples of locomotion mechanisms:

  ▶ Differential-driven mobile robots.

  ▶ Car-like mobile robots.

  ▶ Omni-directional locomotion.
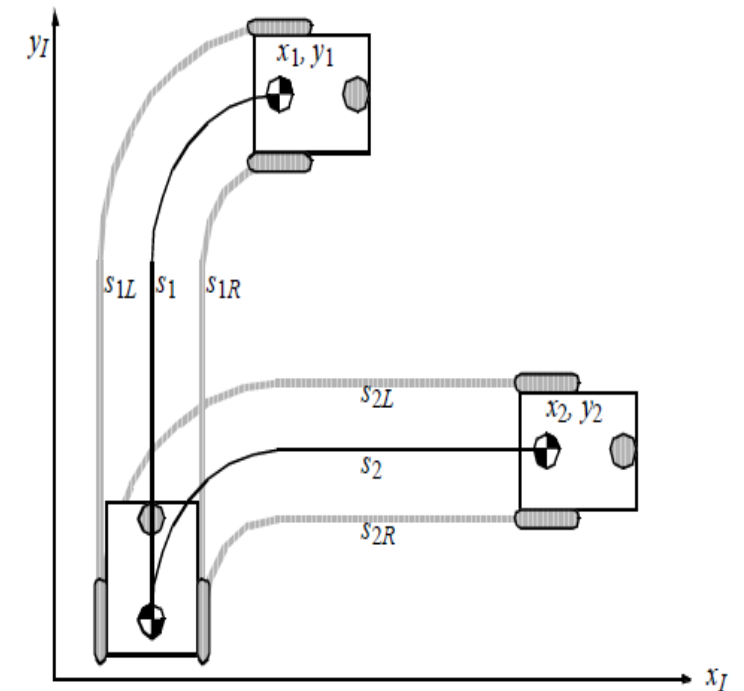
  ▶ Legged robots.

  ▶ Flying Robots.

Mobile Robot Kinematics

▶ Robot Arms vs. Mobile robots:

> ▶ Robot arms are fixed to the ground and usually comprised of a single chain of actuated links.

> ▶ Both are concerned with forward and inverse kinematics.

> ▶ Mobile robots can move unbound with respect to their environment.

>> ▶ There is no direct (=instantaneous) way to measure the robot's position.

>> ▶ Position must be integrated over time, depends on path taken.

> ▶ Understanding mobile robot motion starts with understanding wheel constraints.

Non-Holonomic Systems

▶ Differential equations are not integrable to the final position.

▶ The measure of the traveled distance of each wheel is not sufficient to calculate the final position of the robot. One has also to know how this movement was executed as a function of time.

▶ Note:

  ▶ $S_1 = S_2$, $S_{1L} = S_{2L}$, $S_{1R} = S_{2R}$

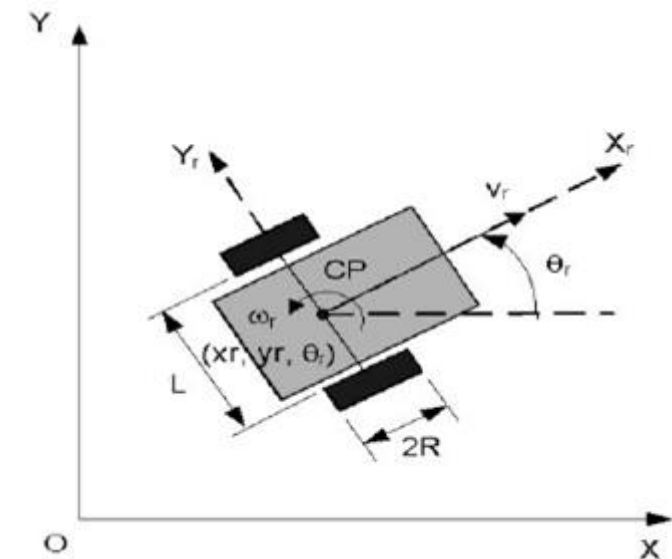 ▶ $x_1 \neq x_2$, $y_1 \neq y_2$

Forward and Inverse Kinematics

- Forward Kinematics:
  - Transformation from joint to physical space.

- Inverse Kinematics:
  - Transformation from physical to joint space.
  - Required for control.

- Due to Non-holonomic constraints in mobile robots:
  - We deal with **differential** (inverse) kinematics.
  - Transformation between velocities instead of positions.

Differential Drive Mobile Robots

DIFFERENTIAL DRIVE LOCOMOTION

FORWARD KINEMATICS MODEL

INVERSE KINEMATICS MODEL

Differential Drive Locomotion

▶ A mobile robot consists of two drive wheels mounted on a common axis, and each wheel can independently being driven either forward or backward.

▶ By varying the velocities of the two wheels, we can vary the trajectories that the robot takes.

▶ Assumptions:

   ▶ Robot pose w.r.t inertial frame $\xi_I = [x_r, y_r, \theta_r]^T$

   ▶ Inertial frame: $\{X, Y\}$, Robot frame: $\{X_r, Y_r\}$

   ▶ Linear velocity: V_r , Angular velocity: ω_r

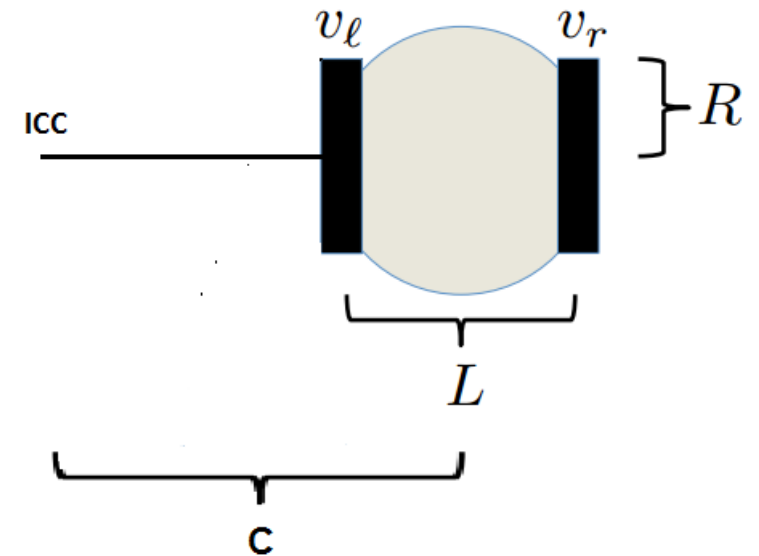   ▶ Right and left wheel linear velocities: v_r , v_l respectively

Differential Drive Kinematics model

▶ Forward kinematics:

    ▶ Expresses robot state in terms of wheel velocities.

    ▶ Robot velocity w.r.t inertial frame $\dot{\xi}_I = [\dot{x}_r, \dot{y}_r, \dot{\theta}_r]^T$

    ▶ Robot velocity w.r.t robot frame $\dot{\xi}_R = [V_r, 0, \omega_r]^T$

        ▶ Note that the linear velocity of robot occurs only in the direction of X_r

    ▶ $\dot{\xi}_I = R(\theta) . \dot{\xi}_R = \begin{bmatrix} c\theta_r & -s\theta_r & 0 \\ s\theta_r & c\theta_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_r \\ 0 \\ \omega_r \end{bmatrix}$

    ▶ Now we need to express V_r and ω_r in terms of wheel velocities v_r and v_l

Differential Drive Kinematics model

▶ Assume $v_r > v_l$ the robot rotates around ICC with angular velocity ω_r

▶ ICC: Instantaneous Center of Curvature.

▶ $v_r = \omega_r(C + \frac{L}{2})$, $v_l = \omega_r(C - \frac{L}{2})$

▶ From the above two equations: $\boxed{\omega_r = \frac{v_r - v_l}{L}}$

▶ Robot linear velocity: $V_r = \omega_r C$, $C = \frac{L(v_r + v_l)}{2(v_r - v_l)}$

▶ $\boxed{V_r = \frac{v_r + v_l}{2}}$

Differential Drive Kinematics model

▶ Inverse Kinematics:

    ▶ Compute the desired wheel velocities to track desired trajectory.

    ▶ Express v_r and v_l in terms of $\dot{\xi}_I$

    ▶ Given: $\dot{\xi}_I(t) = [\dot{x}_r, \dot{y}_r, \dot{\theta}_r]^T$ that achieve a prescribed trajectory.

    ▶ $V_r = \sqrt{\dot{x}_r^2 + \dot{y}_r^2}$, $\omega_r = \dot{\theta}_r$

    ▶ $v_r = V_r + \dfrac{L}{2}\omega_r$

    ▶ $v_l = V_r - \dfrac{L}{2}\omega_r$
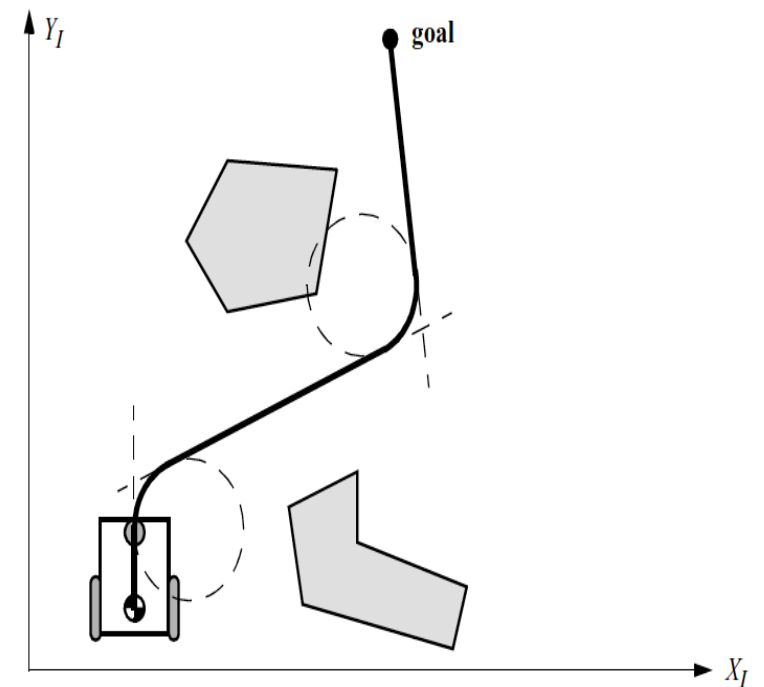
Control of mobile robots

KINEMATIC CONTROL

OPEN-LOOP CONTROL

CLOSED-LOOP CONTROL

Motion Control: (Kinematic Control)

▶ The objective of a kinematic controller is to follow a trajectory described by its position or velocity profile as a function of time.

▶ Motion control is not straight forward because mobile robots are typically non-holonomic and MIMO systems.

▶ Most controllers (including the controllers presented here) are not considering the dynamics of the system (i.e. kinematic controllers).

▶ Two controllers are considered:

   ▶ Open-loop Control (Trajectory-Following).

   ▶ Closed-loop Control (Feedback Controller).

Open-loop Control

▶ Given: A path to follow.

▶ The path is divided into motion segments:

  ▶ Straight lines and segments of circles.

▶ The control problem:

  ▶ Precompute a smooth trajectory based on line and circle segments which drives the robot from the initial position to the final position.

  ▶ Calculate velocities to be applied to robot wheels for certain time frames so that the robot follows the trajectory.
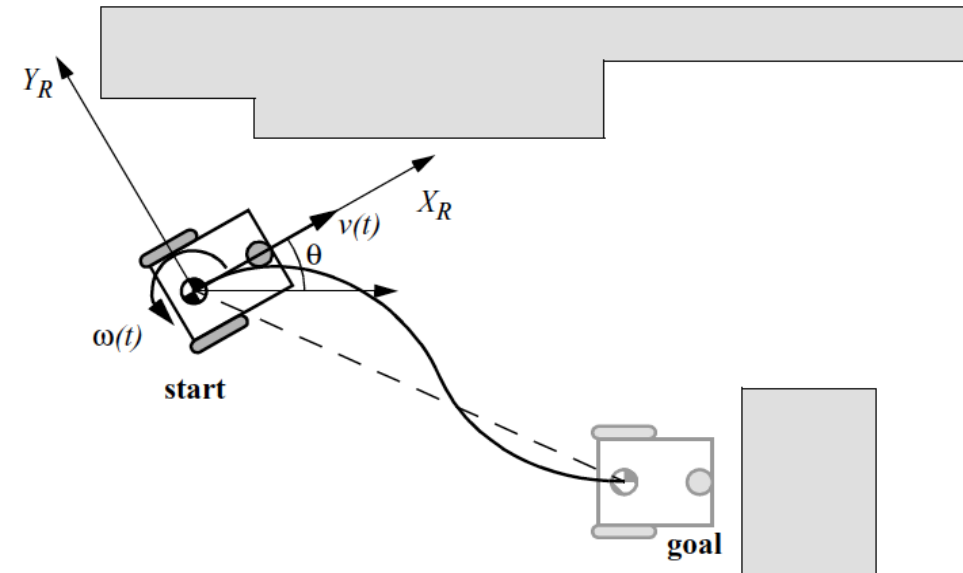
Open-loop Control

▶ Why this approach is regarded as open-loop control ?

   ▶ Measured robot position is not fed back for velocity or position control.

▶ Disadvantages:

   ▶ Limitations and constraints of robot velocities and accelerations.

   ▶ Pre-computing a feasible trajectory is not an easy task.

   ▶ Doesn't support trajectory correction for dynamical change in the environment.

Closed-loop Control

▶ Use MIMO state feedback controller.

▶ With this controller,
the robot path planning task is reduced to
setting intermediate points on the path.

▶ The control objective is to drive the robot
from initial position and orientation to a
predefined goal position and orientation.

Closed-loop Control

▶ The pose error vector given in the robot reference frame is $e^R = [x, y, \theta]^T$ where $x, y, and \ \theta$ are goal coordinates in the robot reference frame.

▶ Find a control matrix K (if exists) such that:

▶ $K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}, k_{ij} = k(t, e)$

▶ The control of v(t) and ω(t): $\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = K.e^R = K.[x, y, \theta]^T$

▶ v(t) and ω(t) are robot linear and angular velocities.

▶ Drives the error e to zero $\lim_{t \to \infty} e(t) = 0$

Reference

- R. Siegwart and I. Nourbakhsh,
  "*Introduction to Autonomous Mobile Robots*",
  The MIT Press, 2004.